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Exact Solutions of Einstein's Field Equations Ordinary Differential Equations and Their Solutions Almost Periodic Solutions of Differential Equations in Banach Spaces Handbook of Exact Solutions for Ordinary Differential Equations Nonlinear Partial Differential Equations Ordinary Differential Equations and Their Solutions The Solution of Equations in Integers Pointwise Bounds for Solutions of the Cauchy Problem for Elliptic Equations Automated Solution of Differential Equations by the Finite Element Method Handbook of Ordinary Differential Equations A Proof of Existence of Particle-like Solutions of Einstein Dirac Equations Generalized Solutions of Operator Equations and Extreme Elements Fractional Partial Differential Equations and Their Numerical Solutions Introductory Differential Equations Handbook of Exact Solutions for Ordinary Differential Equations Almost Global Solutions of Capillary-Gravity Water Waves Equations on the Circle Numerical Solution of Partial Differential Equations—III, SYNSPADE 1975 Boundary Value Problems for Systems of Differential, Difference and Fractional Equations Modelling with Differential and Difference Equations Ordinary Differential Equations Positive Solutions of Differential, Difference and Integral Equations Differential Equation Solutions with MATLAB® Nonlinear Partial Differential Equations in Engineering and Applied Science Harmonic Wave Systems: Partial Differential Equations of the Helmholtz Decomposition PETSc for Partial Differential Equations: Numerical Solutions in C and Python Inverse Problems and Nonlinear Evolution Equations Numerical Solutions of Boundary Value Problems of Non-linear Differential Equations Differential Equations with Linear Algebra Problems and Solutions in Ordinary Differential Equations A Stability Technique for Evolution Partial Differential Equations Recent Advances in Partial Differential Equations, Venice 1996 Handbook of Integral Equations Functional and Impulsive Differential Equations of Fractional Order An Introduction to Difference Equations Introduction to Difference Equations Numerical Solution of Stochastic Differential Equations Soliton Equations and their Algebro-Geometric Solutions: Volume 1, (1+1)-Dimensional Continuous Models Recent Investigations of Differential and Fractional Equations and Inclusions Introduction to Differential Equations with Dynamical Systems Introduction to Multidimensional Integrable Equations

Linearity plays a critical role in the study of elementary differential equations; linear differential equations, especially systems thereof, demonstrate a fundamental application of linear algebra. In Differential Equations with Linear Algebra, we explore this interplay between linear algebra and differential equations and examine introductory and important ideas in each, usually through the lens of important problems that involve differential equations. Written at a sophomore level, the text is accessible to students who have completed multivariable calculus. With a systems-first approach, the book is appropriate for courses for majors in mathematics, science, and engineering that study systems of differential equations. Because of its emphasis on linearity, the text opens with a full chapter devoted to essential ideas in linear algebra. Motivated by future problems in systems of differential equations, the chapter on linear algebra introduces such key ideas as systems of algebraic equations, linear combinations, the eigenvalue problem, and bases and dimension of vector spaces. This chapter enables students to quickly learn enough linear algebra to appreciate the structure of solutions to linear differential equations and systems thereof in subsequent study and to apply these ideas regularly. The book offers an example-driven approach, beginning each chapter with one or two motivating problems that are applied in nature. The following chapter develops the mathematics necessary to solve these problems and explores related topics further. Even in more theoretical developments, we use an example-first style to build intuition and understanding before stating or proving general results. Over 100 figures provide visual demonstration of key ideas; the use of the computer algebra system Maple and Microsoft Excel are presented in detail throughout to provide further perspective and support students' use of technology in solving problems. Each chapter closes with several substantial projects for further study, many of which are based in applications. Errata sheet available at: www.oup.com/us/companion.websites/9780195385861/pdf/errata.pdf Covering applications to physics and engineering as well, this relatively elementary discussion of algebraic equations with integral coefficients and with more than one unknown will appeal to students and mathematicians from high school level onward. 1961 edition. The Portable, Extensible Toolkit for Scientific Computation (PETSc) is an open-source library of advanced data structures and methods for solving linear and nonlinear equations and for managing discretizations. This book uses these modern numerical tools to demonstrate how to solve nonlinear partial differential equations (PDEs) in parallel. It starts from key mathematical concepts, such as Krylov space methods, preconditioning, multigrid, and Newton’s method. In PETSc these components are composed at run time into fast solvers. Discretizations are introduced from the beginning, with an emphasis on finite difference and finite element methodologies. The example C programs of the first 12 chapters, listed on the inside front cover, solve (mostly) elliptic and parabolic PDE problems. Discretization leads to large, sparse, and generally nonlinear systems of algebraic equations. For such problems, mathematical solver concepts are explained and illustrated through the examples, with sufficient context to speed further development. PETSc for Partial Differential Equations addresses both discretizations and fast solvers for PDEs, emphasizing practice more than theory. Well-structured examples lead to run-time choices that result in high solver performance and parallel scalability. The last two chapters build on the reader’s understanding of fast solver concepts when applying the Firedrake Python finite element solver library. This textbook, the first to cover PETSc programming for nonlinear PDEs, provides an on-ramp for graduate students and researchers to a major area of high-performance computing for science and engineering. It is suitable as a supplement for courses in scientific computing or numerical methods for differential equations. A paperback edition of a classic text, this book gives a unique survey of the known solutions of Einstein's field equations for vacuum, Einstein-Maxwell, pure radiation and perfect fluid sources. It introduces the foundations of differential geometry and Riemannian geometry and the methods used to characterize, find or construct solutions. The solutions are then considered, ordered by their symmetry group, their algebraic structure (Petrov type) or other invariant properties such as special subspaces or tensor fields and embedding properties. Includes all the developments in the field since the first edition and contains six completely new chapters, covering topics including generation methods and their application, colliding waves, classification of metrics by invariants and treatments of homothetic motions. This book is an important resource for graduates and researchers in relativity, theoretical physics, astrophysics and mathematics. It can also be used as an introductory text on some mathematical aspects of general relativity. Lax and Nirenberg are two of the most distinguished mathematicians of our times. Their work on partial differential equations (PDEs) over the last half-century has dramatically advanced the subject and has profoundly influenced the course of mathematics. A huge part of the development in PDEs during this period has either been through their work, motivated by it or achieved by their postdocs and students. A large number of mathematicians honored these two exceptional scientists in a week-long conference in Venice (June 1996) on the occasion of their 70th birthdays. This volume contains the proceedings of the conference, which focused on the modern theory of nonlinear PDEs and their applications. Among the topics treated are turbulence, kinetic models of a rarefied gas, vortex filaments, dispersive waves, singular limits and blow-up solutions, conservation laws, Hamiltonian systems and others. The conference served as a forum for the dissemination of new scientific ideas and discoveries and enhanced scientific communication by bringing together such a large number of scientists working in related fields. THe event allowed the international mathematics community to honor two of its outstanding members. This monograph presents recent developments in spectral conditions for the existence of periodic and almost periodic solutions of inhomogenous equations in Banach Spaces. Many of the results represent significant advances in this area. In particular, the authors systematically present a new approach based on the so-called evolution semigroups with an original decomposition technique. The book also extends classical techniques, such as fixed points and stability methods, to abstract functional differential equations with applications to partial functional differential equations. Almost Periodic Solutions of Differential Equations in Banach Spaces will appeal to anyone working in mathematical analysis. The Handbook of Exact Solutions for Ordinary Differential Equations contains a collection of more than 5,000 ordinary differential equations and their solutions. Coverage in this volume includes equations that are of interest to researchers but difficult to integrate (Abel equations, Emden-Fowler equations, Painleve equations, etc.), and equations relevant to applications in heat and mass transfer, nonlinear mechanics, hydrodynamics, nonlinear oscillations, combustion, chemical engineering, and other related fields. In analysing nonlinear phenomena many mathematical models give rise to problems for which only nonnegative solutions make sense. In the last few years this discipline has grown dramatically. This state-of-the-art volume offers the authors' recent work, reflecting some of the major advances in the field as well as the diversity of the subject. Audience: This volume will be of interest to graduate students and researchers in mathematical analysis and its applications, whose work involves ordinary differential equations, finite differences and integral equations. The focus of this book is on algebro-geometric solutions of completely integrable nonlinear partial differential equations in (1+1)-dimensions, also known as soliton equations. Explicitly treated integrable models include the KdV, AKNS, sine-Gordon, and Camassa-Holm hierarchies as well as the classical massive Thirring system. An extensive treatment of the class of algebro-geometric solutions in the stationary as well as time-dependent contexts is provided. The formalism presented includes trace formulas, Dubrovin-type initial value problems, Baker-Akhiezer functions, and theta function representations of all relevant quantities involved. The book uses techniques from the theory of differential equations, spectral analysis, and elements of algebraic geometry (most notably, the theory of compact Riemann surfaces). The presentation is rigorous, detailed, and self-contained, with ample background material provided in various appendices. Detailed notes for each chapter together with an exhaustive bibliography enhance the presentation offered in the main text. Boundary Value Problems for Systems of Differential, Difference and Fractional Equations: Positive Solutions discusses the concept of a differential equation that brings together a set of additional constraints called the boundary conditions. As boundary value problems arise in several branches of math given the fact that any physical differential equation will have them, this book will provide a timely presentation on the topic. Problems involving the wave equation, such as the determination of normal modes, are often stated as boundary value problems. To be useful in applications, a boundary value problem should be well posed. This means that given the input to the problem there exists a unique solution, which depends continuously on the input. Much theoretical work in the field of partial differential equations is devoted to proving that boundary value problems arising from scientific and engineering applications are in fact well-posed. Explains the systems of second order and higher orders differential equations with integral and multi-point boundary conditions Discusses second order difference equations with multi-point boundary conditions Introduces Riemann-Liouville fractional differential equations with uncoupled and coupled integral boundary conditions During the past decades, the subject of calculus of integrals and derivatives of any arbitrary real or complex order has gained considerable popularity and impact. This is mainly due to its demonstrated applications in numerous seemingly diverse and widespread fields of science and engineering. In connection with this, great importance is attached to the publication of results that focus on recent and novel developments in the theory of any types of differential and fractional differential equation and inclusions, especially covering analytical and numerical research for such kinds of equations. This book is a compilation of articles from a Special Issue of Mathematics devoted to the topic of “Recent Investigations of Differential and Fractional Equations and Inclusions”. It contains some theoretical works and approximate methods in fractional differential equations and inclusions as well as fuzzy integrodifferential equations. Many of the papers were supported by the Bulgarian National Science Fund under Project KP-06-N32/7. Overall, the volume is an excellent witness of the relevance of the theory of fractional differential equations. This book aims to introduce some new trends and results on the study of the fractional differential equations, and to provide a good understanding of this field to beginners who are interested in this field, which is the authors' beautiful hope. This book describes theoretical and numerical aspects of the fractional partial differential equations, including the authors' researches in this field, such as the fractional Nonlinear Schrödinger equations, fractional Landau–Lifshitz equations and fractional Ginzburg–Landau equations. It also covers enough fundamental knowledge on the fractional derivatives and fractional integrals, and enough background of the fractional PDEs. Contents:Physics BackgroundFractional Calculus and Fractional Differential EquationsFractional Partial Differential EquationsNumerical Approximations in Fractional CalculusNumerical Methods for the Fractional Ordinary Differential EquationsNumerical Methods for Fractional Partial Differential Equations Readership: Graduate students and researchers in mathematical physics, numerical analysis and computational mathematics. Key Features:This book covers the fundamentals of this field, especially for the beginnersThe book covers new trends and results in this fieldThe book covers numerical results, which will be of broad interests to researchersKeywords:Fractional Partial Differential Equations;Numerical Solutions Exceptionally clear exposition of an important mathematical discipline and its applications to sociology, economics, and psychology. Topics include calculus of finite differences, difference equations, matrix methods, and more. 1958 edition. The book presents in comprehensive detail numerical solutions to boundary value problems of a number of non-linear differential equations. Replacing derivatives by finite difference approximations in these differential equations leads to a system of non-linear algebraic equations which we have solved using Newton’s iterative method. In each case, we have also obtained Euler solutions and ascertained that the iterations converge to Euler solutions. We find that, except for the boundary values, initial values of the 1st iteration need not be anything close to the final convergent values of the numerical solution. Programs in Mathematica 6.0 were written to obtain the numerical solutions. This text is for courses that are typically called (Introductory) Differential Equations, (Introductory) Partial Differential Equations, Applied Mathematics, and Fourier Series. Differential Equations is a text that follows a traditional approach and is appropriate for a first course in ordinary differential equations (including Laplace transforms) and a second course in Fourier series and boundary value problems. Some schools might prefer to move the Laplace transform material to the second course, which is why we have placed the chapter on Laplace transforms in its location in the text. Ancillaries like

Differential Equations with Mathematica and/or Differential Equations with Maple would be recommended and/or required ancillaries. Because many students need a lot of pencil-and-paper practice to master the essential concepts, the exercise sets are particularly comprehensive with a wide range of exercises ranging from straightforward to challenging. Many different majors will require differential equations and applied mathematics, so there should be a lot of interest in an intro-level text like this. The accessible writing style will be good for non-math students, as well as for undergrad classes. This book focuses the solutions of differential equations with MATLAB. Analytical solutions of differential equations are explored first, followed by the numerical solutions of different types of ordinary differential equations (ODEs), as well as the universal block diagram based schemes for ODEs. Boundary value ODEs, fractional-order ODEs and partial differential equations are also discussed. * Introduces a state-of-the-art method for the study of the asymptotic behavior of solutions to evolution partial differential equations. * Written by established mathematicians at the forefront of their field, this blend of delicate analysis and broad application is ideal for a course or seminar in asymptotic analysis and nonlinear PDEs. * Well-organized text with detailed index and bibliography, suitable as a course text or reference volume. Numerical Solution of Partial Differential Equations—III: Synspade 1975 provides information pertinent to those difficult problems in partial differential equations exhibiting some type of singular behavior. This book covers a variety of topics, including the mathematical models and their relation to experiment as well as the behavior of solutions of the partial differential equations involved. Organized into 16 chapters, this book begins with an overview of elastodynamic results for stress intensity factors of a bifurcating crack. This text then discusses the effects of nonlinearities, such as bifurcation, which occur in problems of nonlinear mechanics. Other chapters consider the equations of changing type and those with rapidly oscillating coefficients. This book discusses as well the effective computational methods for numerical solutions. The final chapter deals with the principal results on G-convergence, such as the convergence of the Green's operators for Dirichlet's and other boundary problems. This book is a valuable resource for engineers and mathematicians. In this volume are twenty-eight papers from the Conference on Nonlinear Partial Differential Equationsin Engineering and Applied Science, sponsored by the Office of Naval Research and held at the Universityof Rhode Island in June, 1979. Included are contributions from an international group of distinguishedmathematicians, scientists, and engineers coming from a wide variety of disciplines and having a commoninterest in the application of mathematics, particularly nonlinear partial differential equations, to realworld problems.The subject matter ranges from almost purely mathematical topics in numerical analysis and bifurcationtheory to a host of practical applications that involve nonlinear partial differential equations, suchas fluid dynamics, nonlinear waves, elasticity, viscoelasticity, hyperelasticity, solitons, metallurgy, shocklessairfoil design, quantum fields, and Darcy's law on flows in porous media.Non/inear Partial Differential Equations in Engineering and Applied Science focuses on a variety oftopics of specialized, contemporary concern to mathematicians, physical and biological scientists, andengineers who work with phenomena that can be described by nonlinear partial differential equations. Unparalleled in scope compared to the literature currently available, the Handbook of Integral Equations, Second Edition contains over 2,500 integral equations with solutions as well as analytical and numerical methods for solving linear and nonlinear equations. It explores Volterra, Fredholm, WienerHopf, Hammerstein, Uryson, and other equa Exact solutions of differential equations continue to play an important role in the understanding of many phenomena and processes throughout the natural sciences in that they can verify the correctness of or estimate errors in solutions reached by numerical, asymptotic, and approximate analytical methods. The new edition of this bestselling handbook now contains the exact solutions to more than 6200 ordinary differential equations. The authors have made significant enhancements to this edition, including: An introductory chapter that describes exact, asymptotic, and approximate analytical methods for solving ordinary differential equations The addition of solutions to more than 1200 nonlinear equations An improved format that allows for an expanded table of contents that makes locating equations of interest more quickly and easily Expansion of the supplement on special functions This handbook's focus on equations encountered in applications and on equations that appear simple but prove particularly difficult to integrate make it an indispensable addition to the arsenals of mathematicians, scientists, and engineers alike. This book is based on the method of operator identities and related theory of S-nodes, both developed by Lev Sakhnovich. The notion of the transfer matrix function generated by the S-node plays an essential role. The authors present fundamental solutions of various important systems of differential equations using the transfer matrix function, that is, either directly in the form of the transfer matrix function or via the representation in this form of the corresponding Darboux matrix, when Bäcklund–Darboux transformations and explicit solutions are considered. The transfer matrix function representation of the fundamental solution yields solution of an inverse problem, namely, the problem to recover system from its Weyl function. Weyl theories of selfadjoint and skew-selfadjoint Dirac systems, related canonical systems, discrete Dirac systems, system auxiliary to the N-wave equation and a system rationally depending on the spectral parameter are obtained in this way. The results on direct and inverse problems are applied in turn to the study of the initial-boundary value problems for integrable (nonlinear) wave equations via inverse spectral transformation method. Evolution of the Weyl function and solution of the initial-boundary value problem in a semi-strip are derived for many important nonlinear equations. Some uniqueness and global existence results are also proved in detail using evolution formulas. The reading of the book requires only some basic knowledge of linear algebra, calculus and operator theory from the standard university courses. Any student wishing to solve problems via mathematical modelling will find that this book provides an excellent introduction to the subject. This treatment presents most of the methods for solving ordinary differential equations and systematic arrangements of more than 2,000 equations and their solutions. The material is organized so that standard equations can be easily found. Plus, the substantial number and variety of equations promises an exact equation or a sufficiently similar one. 1960 edition. An analysis is presented which deals with a technique for approximating the solution to a Cauchy problem for a geneal second-order elliptic patil differential equation defined in an N-dimensional region D. The method is based upon the determination of an a priori bound for the value of an arbitrary function u at a point P in D in terms of the values of u and its gradient on the cauchy surface andA FUNCTIONAL OF THE ELLIPTIC OPERATOR APPLIED TO U. (Author). The Handbook of Ordinary Differential Equations: Exact Solutions, Methods, and Problems, is an exceptional and complete reference for scientists and engineers as it contains over 7,000 ordinary differential equations with solutions. This book contains more equations and methods used in the field than any other book currently available. Included in the handbook are exact, asymptotic, approximate analytical, numerical symbolic and qualitative methods that are used for solving and analyzing linear and nonlinear equations. The authors also present formulas for effective construction of solutions and many different equations arising in various applications like heat transfer, elasticity, hydrodynamics and more. This extensive handbook is the perfect resource for engineers and scientists searching for an exhaustive reservoir of information on ordinary differential equations. Abstract models for many problems in science and engineering take the form of an operator equation. The resolution of these problems often requires determining the existence and uniqueness of solutions to these equations. "Generalized Solutions of Operator Equations and Extreme Elements" presents recently obtained results in the study of the generalized solutions of operator equations and extreme elements in linear topological spaces. The presented results offer new methods of identifying these solutions and studying their properties. These new methods involve the application of a priori estimations and a general topological approach to construct generalized solutions of linear and nonlinear operator equations. The monograph is intended for mathematicians, graduate students and researchers studying functional analysis, operator theory, and the theory of optimal control. This book grew out of lecture notes I used in a course on difference equations that I taught at Trinity University for the past five years. The classes were largely pop ulated by juniors and seniors majoring in Mathematics, Engineering, Chemistry, Computer Science, and Physics. This book is intended to be used as a textbook for a course on difference equations at the level of both advanced undergraduate and beginning graduate. It may also be used as a supplement for engineering courses on discrete systems and control theory. The main prerequisites for most of the material in this book are calculus and linear algebra. However, some topics in later chapters may require some rudiments of advanced calculus. Since many of the chapters in the book are independent, the instructor has great flexibility in choosing topics for the first one-semester course. A diagram showing the interdependence of the chapters in the book appears following the preface. This book presents the current state of affairs in many areas such as stability, Z-transform, asymptoticity, oscillations and control theory. However, this book is by no means encyclopedic and does not contain many important topics, such as Numerical Analysis, Combinatorics, Special functions and orthogonal polyno mials, boundary value problems, partial difference equations, chaos theory, and fractals. The nonselection of these topics is dictated not only by the limitations imposed by the elementary nature of this book, but also by the research interest (or lack thereof) of the author. Skillfully organized introductory text examines origin of differential equations, then defines basic terms and outlines the general solution of a differential equation. Subsequent sections deal with integrating factors; dilution and accretion problems; linearization of first order systems; Laplace Transforms; Newton's Interpolation Formulas, more. The numerical analysis of stochastic differential equations (SDEs) differs significantly from that of ordinary differential equations. This book provides an easily accessible introduction to SDEs, their applications and the numerical methods to solve such equations. From the reviews: "The authors draw upon their own research and experiences in obviously many disciplines... considerable time has obviously been spent writing this in the simplest language possible." --ZAMP This book is a tutorial written by researchers and developers behind the FEniCS Project and explores an advanced, expressive approach to the development of mathematical software. The presentation spans mathematical background, software design and the use of FEniCS in applications. Theoretical aspects are complemented with computer code which is available as free/open source software. The book begins with a special introductory tutorial for beginners. Following are chapters in Part I addressing fundamental aspects of the approach to automating the creation of finite element solvers. Chapters in Part II address the design and implementation of the FEnicS software. Chapters in Part III present the application of FEniCS to a wide range of applications, including fluid flow, solid mechanics, electromagnetics and geophysics. The soliton represents one ofthe most important ofnonlinear phenomena in modern physics. It constitutes an essentially localizedentity with a set ofremarkable properties. Solitons are found in various areas of physics from gravitation and field theory, plasma physics, and nonlinear optics to solid state physics and hydrodynamics. Nonlinear equations which describe soliton phenomena are ubiquitous. Solitons and the equations which commonly describe them are also of great mathematical interest. Thus, the dis covery in 1967and subsequent development ofthe inversescattering transform method that provides the mathematical structure underlying soliton theory constitutes one of the most important developments in modern theoretical physics. The inversescattering transform method is now established as a very powerful tool in the investigation of nonlinear partial differential equations. The inverse scattering transform method, since its discoverysome two decades ago, has been applied to a great variety of nonlinear equations which arise in diverse fields of physics. These include ordinary differential equations, partial differential equations, integrodifferential, and differential-difference equations. The inverse scattering trans form method has allowed the investigation of these equations in a manner comparable to that of the Fourier method for linear equations. The book presents qualitative results for different classes of fractional equations, including fractional functional differential equations, fractional impulsive differential equations, and fractional impulsive functional differential equations, which have not been covered by other books. It manifests different constructive methods by demonstrating how these techniques can be applied to investigate qualitative properties of the solutions of fractional systems. Since many applications have been included, the demonstrated techniques and models can be used in training students in mathematical modeling and in the study and development of fractional-order models. This work will serve as an excellent first course in modern analysis. The main focus is on showing how self-similar solutions are useful in studying the behavior of solutions of nonlinear partial differential equations, especially those of parabolic type. This textbook will be an excellent resource for self-study or classroom use. Many textbooks on differential equations are written to be interesting to the teacher rather than the student. Introduction to Differential Equations with Dynamical Systems is directed toward students. This concise and up-to-date textbook addresses the challenges that undergraduate mathematics, engineering, and science students experience during a first course on differential equations. And, while covering all the standard parts of the subject, the book emphasizes linear constant coefficient equations and applications, including the topics essential to engineering students. Stephen Campbell and Richard Haberman--using carefully worded derivations, elementary explanations, and examples, exercises, and figures rather than theorems and proofs--have written a book that makes learning and teaching differential equations easier and more relevant. The book also presents elementary dynamical systems in a unique and flexible way that is suitable for all courses, regardless of length. The goal of this monograph is to prove that any solution of the Cauchy problem for the capillary-gravity water waves equations, in one space dimension, with periodic, even in space, small and smooth enough initial data, is almost globally defined in time on Sobolev spaces, provided the gravity-capillarity parameters are taken outside an exceptional subset of zero measure. In contrast to the many results known for these equations on the real line, with decaying Cauchy data, one cannot make use of dispersive properties of the linear flow. Instead, a normal forms-based procedure is used, eliminating those contributions to the Sobolev energy that are of lower degree of homogeneity in the solution. Since the water waves equations form a quasi-linear system, the usual normal forms approaches would face the well-known problem of losses of derivatives in the unbounded transformations. To overcome this, after a paralinearization of the capillary-gravity water waves equations, we perform several paradifferential reductions to obtain a diagonal system with constant coefficient symbols, up to smoothing remainders. Then we start with a normal form procedure where the small divisors are compensated by the previous paradifferential regularization. The reversible structure of the water waves equations, and the fact that we seek solutions even in space, guarantees a key cancellation which prevents the growth of the Sobolev norms of the solutions. Harmonic Wave Systems is the first textbook about the computational method of Decomposition in Invariant Structures (DIS) that generalizes the analytical methods of separation of variables, undetermined coefficients, asymptotic expansions, and series expansions. In recent years, there has been a boom in publications on propagation of nonlinear waves described by a fascinating list of partial differential equations (PDEs). The vast majority of wave problems are reducible to one-dimensional ones in propagation variables. However, a list of publications with two- and three-dimensional applications of the DIS method is brief. The book offers a comprehensive and rigorous treatment of the DIS method in two and three dimensions using the PDE approach to the Helmholtz decomposition that provides the most general background for mathematical modelling of harmonic waves in fluid dynamics, electrodynamics, heat transfer, and other numerous areas of science and engineering, which are dealing with propagation and interaction of N internal waves.